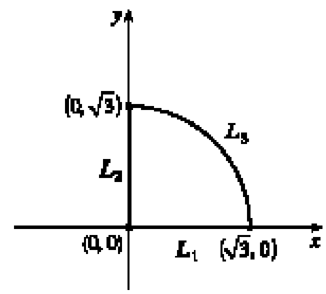
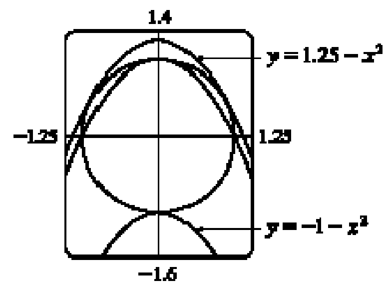
11.7: 30; 11.8: 2, 18, 40(b)



30. $f_x = y^2$ and $f_y = 2xy$, and since $f_x = 0 \iff y = 0$, there are no critical points in the interior of D. Along L_1, $y = 0$ and $f(x, 0) = 0$. Along L_2, $x = 0$ and $f(0, y) = 0$. Along L_3, $y = \sqrt{3 - x^2}$, so let $g(x) = f\left(x, \sqrt{3 - x^2}\right) = 3x - x^3$ for $0 \le x \le \sqrt{3}$. Then $g'(x) = 3 - 3x^2 = 0 \iff x = 1$. The maximum value is $f\left(1, \sqrt{2}\right) = 2$ and the minimum occurs both at $x = 0$ and $x = \sqrt{3}$ where $f\left(0, \sqrt{3}\right) = f\left(\sqrt{3}, 0\right) = 0$. Thus the absolute maximum of f on D is $f\left(1, \sqrt{2}\right) = 2$, and the absolute minimum is 0 which occurs at all points along L_1 and L_2.



2. (a) The values $c = \pm 1$ and $c = 1.25$ seem to give curves which are tangent to the circle. These values represent possible extreme values of the function $x^2 + y$ subject to the constraint $x^2 + y^2 = 1$.

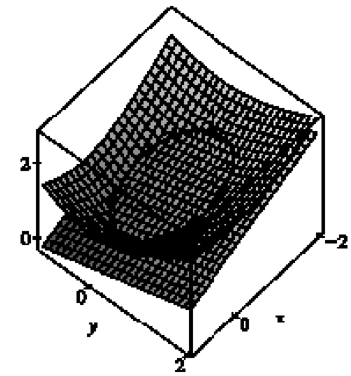
(b) $\nabla f = \langle 2x, 1 \rangle$, $\lambda \nabla g = \langle 2\lambda x, 2\lambda y \rangle$. So $2x = 2\lambda x \Rightarrow$ either $\lambda = 1$ or $x = 0$. If $\lambda = 1$, then $y = \frac{1}{2}$ and so $x = \pm\frac{\sqrt{3}}{2}$ (from the constraint). If $x = 0$, then $y = \pm 1$. Therefore f has possible extreme values at the points $(0, \pm 1)$ and $\left(\pm\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$. We calculate $f\left(\pm\frac{\sqrt{3}}{2}, \frac{1}{2}\right) = \frac{5}{4}$ (the maximum value), $f(0, 1) = 1$, and $f(0, -1) = -1$ (the minimum value). These are our answers from (a).

18. $f(x, y) = 2x^2 + 3y^2 - 4x - 5 \Rightarrow \nabla f = \langle 4x - 4, 6y \rangle = \langle 0, 0 \rangle \Rightarrow x = 1, y = 0$. Thus $(1, 0)$ is the only critical point of f, and it lies in the region $x^2 + y^2 < 16$. On the boundary, $g(x, y) = x^2 + y^2 = 16 \Rightarrow \lambda \nabla g = \langle 2\lambda x, 2\lambda y \rangle$, so $6y = 2\lambda y \Rightarrow$ either $y = 0$ or $\lambda = 3$. If $y = 0$, then $x = \pm 4$; if $\lambda = 3$, then $4x - 4 = 2\lambda x \Rightarrow x = -2$ and $y = \pm 2\sqrt{3}$. Now $f(1, 0) = -7$, $f(4, 0) = 11$, $f(-4, 0) = 43$, and $f\left(-2, \pm 2\sqrt{3}\right) = 47$. Thus the maximum value of $f(x, y)$ on the disk $x^2 + y^2 \le 16$ is $f\left(-2, \pm 2\sqrt{3}\right) = 47$, and the minimum value is $f(1, 0) = -7$.

40. (a) Parametric equations for the ellipse are easiest to determine using cylindrical coordinates. The cone is given by $z = r$, and the plane is $4r\cos\theta - 3r\sin\theta + 8z = 5$. Substituting $z = r$ into the plane equation gives $4r\cos\theta - 3r\sin\theta + 8r = 5 \quad \Rightarrow$

$r = \dfrac{5}{4\cos\theta - 3\sin\theta + 8}$. Since $z = r$ on the ellipse, parametric equations (in cylindrical coordinates) are

$\theta = t, r = z = \dfrac{5}{4\cos t - 3\sin t + 8}, 0 \le t \le 2\pi.$



(b) We need to find the extreme values of $f(x,y,z) = z$ subject to the two constraints $g(x,y,z) = 4x - 3y + 8z = 5$ and $h(x,y,z) = x^2 + y^2 - z^2 = 0$. $\nabla f = \lambda \nabla g + \mu \nabla h \quad \Rightarrow \quad \langle 0,0,1\rangle = \lambda\langle 4,-3,8\rangle + \mu\langle 2x, 2y, -2z\rangle$, so we need (1) $4\lambda + 2\mu x = 0 \quad \Rightarrow \quad x = -\frac{2\lambda}{\mu}$, (2) $-3\lambda + 2\mu y = 0 \quad \Rightarrow \quad y = \frac{3\lambda}{2\mu}$, (3) $8\lambda - 2\mu z = 1 \quad \Rightarrow \quad z = \frac{8\lambda - 1}{2\mu}$, (4) $4x - 3y + 8z = 5$, and (5) $x^2 + y^2 = z^2$. [Note that $\mu \neq 0$, else $\lambda = 0$ from (1), but substitution into (3) gives a contradiction.] Substituting (1), (2), and (3) into (4) gives $4\left(-\frac{2\lambda}{\mu}\right) - 3\left(\frac{3\lambda}{2\mu}\right) + 8\left(\frac{8\lambda-1}{2\mu}\right) = 5 \quad \Rightarrow \quad \mu = \frac{39\lambda - 8}{10}$ and into (5) gives $\left(-\frac{2\lambda}{\mu}\right)^2 + \left(\frac{3\lambda}{2\mu}\right)^2 = \left(\frac{8\lambda-1}{2\mu}\right)^2 \quad \Rightarrow \quad 16\lambda^2 + 9\lambda^2 = (8\lambda - 1)^2 \quad \Rightarrow \quad 39\lambda^2 - 16\lambda + 1 = 0 \quad \Rightarrow \quad \lambda = \frac{1}{13}$ or $\lambda = \frac{1}{3}$. If $\lambda = \frac{1}{13}$ then $\mu = -\frac{1}{2}$ and $x = \frac{4}{13}, y = -\frac{3}{13}, z = \frac{5}{13}$. If $\lambda = \frac{1}{3}$ then $\mu = \frac{1}{2}$ and $x = -\frac{4}{3}, y = 1, z = \frac{5}{3}$. Thus the highest point on the ellipse is $\left(-\frac{4}{3}, 1, \frac{5}{3}\right)$ and the lowest point is $\left(\frac{4}{13}, -\frac{3}{13}, \frac{5}{13}\right)$.